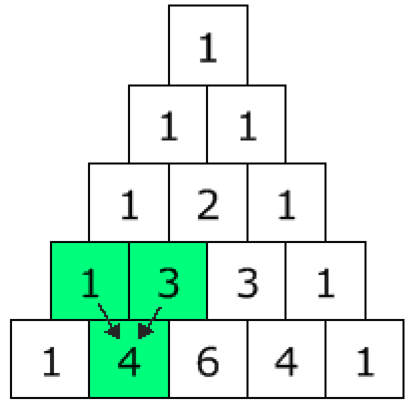
Pascal's Triangle

One of the most interesting Number Patterns is Pascal's Triangle (named after *Blaise Pascal*, a famous French Mathematician and Philosopher).

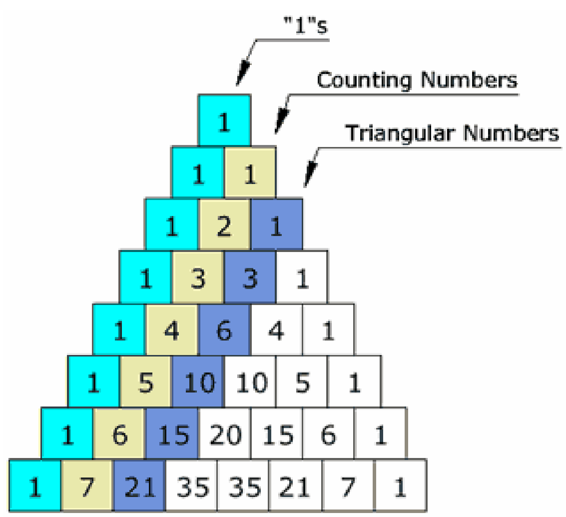
To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

Each number is just the two numbers above it added together (except for the edges, which are all "1").

(Here I have highlighted that **1+3 = 4)**



Patterns Within the Triangle



Diagonals

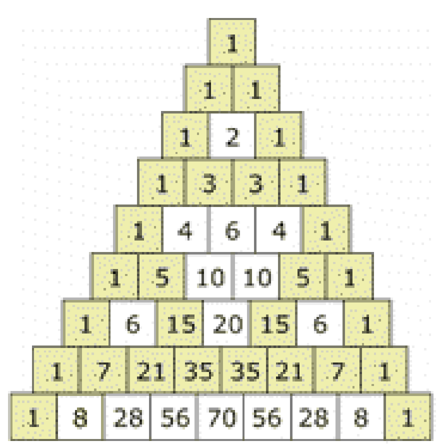
The first diagonal is, of course, just "1"s, and the next diagonal has the Counting Numbers (1,2,3, etc).

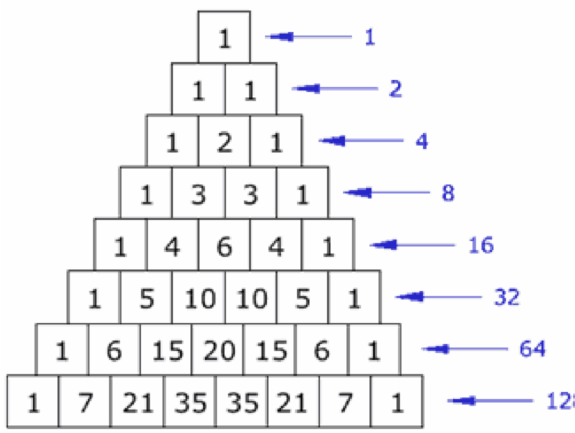
The third diagonal has the triangular numbers

(The fourth diagonal, not highlighted, has the tetrahedral numbers.)

Odds and Evens

If you color the Odd and Even numbers, you end up with a pattern the same as the Sierpinski Triangle





Horizontal Sums

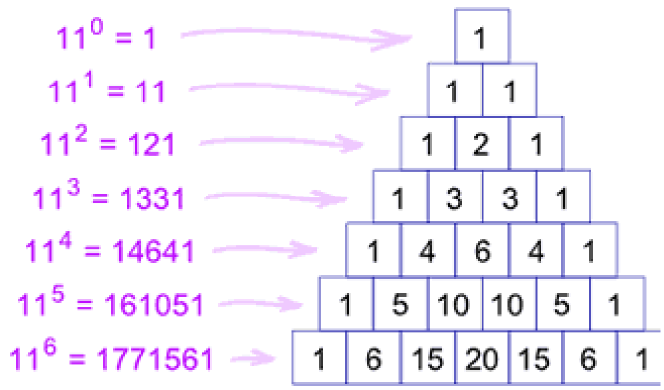
What do you notice about the horizontal sums?

Is there a pattern? Isn't it amazing! It doubles each time (powers of 2).

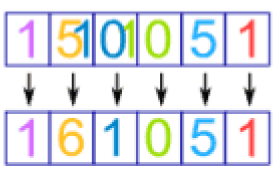
Exponents of 11

Each line is also the powers (exponents) of 11:

- 11^0=1 (the first line is just a "1")
- 11^1=11 (the second line is "1" and "1")
- 11^2=121 (the third line is "1", "2", "1")
- etc!



But what happens with 11^5? Simple! The digits just overlap, like this:

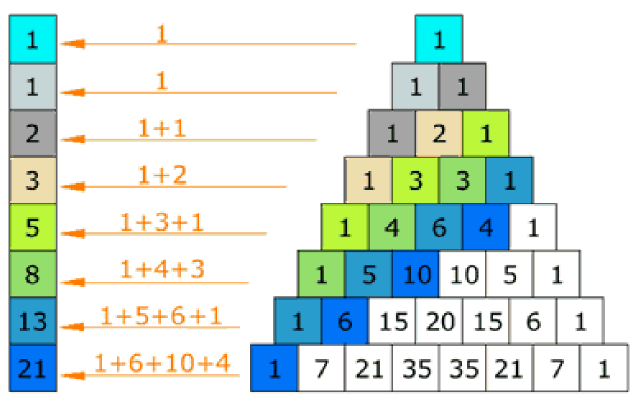


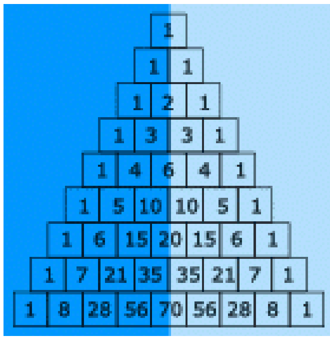
The same thing happens with 11^6 etc.

Fibonacci Sequence

Try this: make a pattern by going up and then along, then add up the values (as illustrated) ... you will get the Fibonacci Sequence.

(The Fibonacci Sequence starts "1, 1" and then continues by adding the two previous numbers, for example 3+5=8, then 5+8=13, etc)





Symmetrical

And the triangle is also symmetrical. The numbers on the left side have identical matching numbers on the right side, like a mirror image.

Using Pascal's Triangle

Probability (Heads and Tails)

Pascal's Triangle can show you how many ways heads and tails can combine. This can then show you "the odds" (or probability) of any combination.

For example, if you toss a coin three times, there is only one combination that will give you three heads (HHH), but there are three that will give two heads and one tail (HHT, HTH, THH), also three that give one head and two tails (HTT, THT, TTH) and one for all Tails (TTT). This is the pattern "1,3,3,1" in Pascal's Triangle.

Tosses	Possible Results (Grouped)	Pascal's Triangle
1	H T	1, 1
2	HH HT TH TT	1, 2, 1
3	HHH HHT, HTH, THH HTT, THT, TTH TTT	1, 3, 3, 1
4	HHHH HHHT, HHTH, HTHH, THHH HHTT, HTHT, HTTH, THHT, THTH, TTHH HTTT, THTT, TTHT, TTTH TTTT	1, 4, 6, 4, 1
	... etc ...	

Example: What is the probability of getting exactly two heads with 4 coin tosses?

There are 1+4+6+4+1 = 16 (or 2^4=16) possible results, and 6 of them give exactly two heads. So the probability is 6/16, or 37.5%.

Combinations

The triangle also shows you how many Combinations of objects are possible.

Example: You have 16 pool balls. How many different ways could you choose just 3 of them (ignoring the order that you select them)?

Answer: go down to row 16 (the top row is 0), and then along 3 places and the value there is your answer, **560**.

Here is an extract at row 16:

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          1    14    91    364  ...
       1    15    105   455   1365  ...
     1    16   120   560   1820  4368  ...
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A Formula for Any Entry in The Triangle

In fact there is a formula from Combinations for working out the value at any place in Pascal's triangle:

It is commonly called "n choose k" and written like this: $$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Notation: "n choose k" can also be written **C(n,k)**, $\mathbf{^{n}C_{k}}$ or even $\mathbf{_{n}C_{k}}$.

The "!" is "factorial" and means to multiply a series of descending natural numbers. Examples:

- 4! = 4 × 3 × 2 × 1 = 24
- 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040
- 1! = 1